## The Value-Price Transformation Formula

(1981)

The transformation of values into prices is one of the favorite subjects in debates on Marx's economics. The problem of transformation is solved by Marx himself by his deduction of the price-form out of the notion of commodity.
We would like to stop that absurd debate on transformation by reconstructing Marx's argumentation in an exact and formalized way.

## I

A commodity is, as known, a value and a use-value, a matter of use ( $U$ ) and a matter of value ( $V$ ). The notion of commodity is the unit of the elements $U$ and $V$, the representation of this notion is $(\mathrm{U}, \mathrm{V})$. The commodity $(\mathrm{U}, \mathrm{V})$ is the result of commodityproducing work. This special work is work (W, work or concrete labour) in producing the useful thing $U$ and labour ( $L$, abstract labour) in creating the value V. Therefore, commodity-producing work is ( $\mathrm{W}, \mathrm{L}$ ) and the whole process of a commodity's production can be noted as follows:
$(1)((W, L) \Rightarrow(U, V))$ or $((W \rightarrow U),(L \rightarrow V))$, i.e. work produces the useful thing and labour produces its value.
Work is as various as are the useful things it makes, but labour, value-producing, is nothing other than the measurement of work in terms of time, or work-time, which due to historical circumstances has been treated as synonymous with labour. Thus, a theory of value and a theory of labour are the same problem. The quantity of labour is identical to the quantity of value and therefore the question is: to what extent does work-time count as labour?

Not all real work-time counts as labour. A given work-time quantifies labour in a one-toone ratio only when it is socially necessary work-time $\mathrm{t}_{\text {nec }}(\mathrm{W})$ which is tested by the market. Socially necessary work-time is first average work, which has in a given society an average intensity, a general technological and natural standard and a normal degree of skill and organization of manpower. Second, socially necessary work-time is needed work; that means the commodities must be demanded in exactly the quantity they are produced. What work is needed will be decided a priori in a household or a posteriori by a market. Socially necessary work-time is labour, when the market has decided in what quantity work has been needed. Suppose that all commodities are products of average work, but their quantities are too small; then they realize an overaverage work-time as labour, i.e. value - and vice versa. The formula of labour Limplies the Marxian concept of value V and the existence of the market:
(2) $L:=t_{n e c}(W)$, (that means: labour is defined as socially necessary work-time, market-realized).
Assuming that labour or value exists, the determinations of average work can be expressed by two functions: intensity of work $I(W)$ and productivity of work $P(W)$ :
(3) $\mathrm{I}(\mathrm{W}):=\mathrm{L} / \mathrm{t}(\mathrm{W})$ (labour/work-time ratio),
(4) $\mathrm{P}(\mathrm{W}):=\mathrm{U} / \mathrm{t}(\mathrm{W})$ (product/work-time ratio, intensity presumed to be constant).

Productivity of work is determined by the technical and natural standards and the average skill and organization of manpower in a given society. Changes in a work's productivity alter a commodity's value in the opposite sense and keep constant the total value of the work-time period. Changes in the intensity of work do not touch the value of the single commodity, but the whole value of the work-time period alters in the same direction.

Now, up to this point in the sphere of production, we do not know any prices, only values. The real amount of values can be decided in the sphere of circulation only. Now the different commodities - $(\mathrm{U}, \mathrm{V})_{1},(\mathrm{U}, \mathrm{V})_{2}, \ldots,(\mathrm{U}, \mathrm{V})_{\mathrm{n}}$ or $(\mathrm{U}, \mathrm{V})_{1 \ldots \mathrm{n}}$ - appear on the market. Commodities shall be exchanged. Therefore every unsold commodity on the market takes on a special relation to other commodities. This relation is the value of exchange or form of value. This form is a real act of definition, which makes concrete one commodity's value in terms of the use-value of another commodity. The simple form of value is the definition of one commodity's value $V_{1}$ in the use-value $U_{2}$ of another commodity; the definition of one commodity's value $V_{1}$ in all the use-values $U_{2 \ldots n}$ of all the other commodities on a given market is called total form of value; the definition of all the commodities' values $V_{1 \ldots n}$ in only one use-value $U_{0}$ is the general form of value:
(5) $\left(\mathrm{V}_{1}:=\mathrm{U}_{2}\right) \quad$ (simple form of value),
(6) $\left(\mathrm{V}_{1}:=\mathrm{U}_{2 \ldots \mathrm{n}}\right) \quad$ (total form of value),
(7) $\left(\mathrm{V}_{1 \ldots \mathrm{n}}:=\mathrm{U}_{0}\right) \quad$ (general form of value).

The reconstruction of a simple form after the general form of value is ruling the market results in the price-form:
(8) $\left(V_{1}:=U_{0}\right)$.

The price-form transforms the value into a price. The price always has natural form; it is a useful thing. The price is a definiens of the value. When prices of an article change from day to day and from place to place, it means that the value changes. Changes of the value in general have their source in changes of needed work, not in average worktime. Sometimes, of course, there are variations in the average work-time, and therefore in labour, value and price. But normally conditions of work are fixed and labour alone is variable, like the payable needs are.
We resume: formula (8), the price-form, solves the transformation problem between values and prices. In a wider sense, all the forms of value are formulas of transformation. They translate a value into a use-value, and every use-value in the
market wants to be a price, especially the price of a money's value $\mathrm{V}_{0}$.
There is no problem in calculating the values but it is true that one cannot know how much of the work-time incorporated in commodities is socially necessary before the price-form is reached and the value realized in a definite quantity of the general equivalent $U_{0}$.

The transformation problem reappears on the categorial level of general annual profitrate as a modified price-form, which defines the price of production ' $\mathrm{V}_{1}$ in the marketprice ' $\mathrm{U}_{0}$. The price of production is the modified value ${ }^{\prime} \mathrm{V}$ of a capital-resulted commodity. The cost-price, which is not a price but the incorporated manpower and means of production value $\left(V_{v}+V_{0}\right)$, summed with the general annual profit on it is the price of production. To understand exactly this modified value we will demonstrate the categorial formulas of the general annual profit-rate.

The cost-price per se can be considered as part of a commodity's value and as total value of invested capital. Given the cost-price or value's amount of a capital $\mathrm{C}_{1}=\mathrm{V}_{\mathrm{v}}+\mathrm{V}_{\mathrm{c}}$, its turnover period $t_{u}\left(C_{1}\right)$, its profit or surplus-value $V_{s}\left(C_{1}\right)$, and its turnover number $u\left(C_{1}\right)$, then the annual profit-rate of capital $C_{1}$ is:
(9) $\left(u V_{s} / C_{1}\right)$ or $\left(u V_{s} / V_{v}+V_{0}\right)$.

The annual profit-rate of capital $C_{1}$ rises, if 1) its turnover number rises ${ }^{\text {}} \mathrm{u}$ and therefore the turnover period falls ${ }^{>} t_{u}, 2$ ) the value of the production means falls ${ }^{>} V_{c}, 3$ ) the value of manpower falls ${ }^{>} V_{v}$ and 4) the surplus-value rises ${ }^{<} V_{s}$. The last case may result from a rising exploitation-rate ${ }^{<}\left(\mathrm{V}_{\mathrm{s}} / \mathrm{V}_{\mathrm{v}}\right)$ or from a decreasing organic composition ${ }^{>}\left(\mathrm{V}_{\mathrm{c}} / \mathrm{V}_{\mathrm{v}}\right)$. In formula (9), under the condition of a fixed amount of capital $C_{1}$, a rise in the annual profit-rate as a result of decreasing organic composition and therefore increasing surplusvalue appears as ${ }^{=}\left({ }^{<} V_{v}+{ }^{>} V_{c}\right)$.
Now we will modify the annual profit-rate of capital $C_{1}$ into the general annual profit-rate of all capitals. Assume that the total social capital is divided into two branch-capitals $\mathrm{C}_{1}$ and $C_{2}$. The branch profit-rates are different when $u, V_{s}, V_{v}$ or $V_{c}$ differ in any nonproportional and non-compensatory way. The general annual profit-rate $\varnothing\left(u V_{s} / C\right)$ will be established by competition of investment. If there is a lower profit-rate in branch $\mathrm{C}_{1}$ than in branch $C_{2}$, then $C_{1}$ falls and $C_{2}$ rises given the condition of a fixed total amount of the social capital. That over-investment in capital-branch $\mathrm{C}_{2}$ results in a rising quantity of commodities' output ${ }^{<} x(U, V)_{2}$. The opposite output has the under-investment in the branch $C_{1}$. Prices in branch $C_{1}$ are rising and falling in $C_{2}$; just as are the profit-rates:


An individual capital $C_{1}$, i.e. its cost-price, multiplied by the general annual rate of profit equals its general profit $\emptyset\left(u V_{s}(C)\right)$. The capital's price of production ${ }^{\prime} V\left(C_{1}\right)$ is the sum of its cost-price and its general profit:
(11) ${ }^{\prime} \mathrm{V}\left(\mathrm{C}_{1}\right)=\left(\mathrm{V}_{\mathrm{v}}+\mathrm{V}_{\mathrm{c}}\right)\left(\mathrm{C}_{1}\right)+\varnothing\left(\mathrm{u}_{\mathrm{s}}\left(\mathrm{C}_{1}\right)\right)$.

A commodity's price of production ' $\mathrm{V}_{1}$ results from the capital's price of production divided by the number of the same sort of commodities annually produced by this capital:
(12) ${ }^{\prime} \mathrm{V}_{1}={ }^{\mathrm{V}} \mathrm{V}\left(\mathrm{C}_{1}\right) / \mathrm{x}(\mathrm{U}, \mathrm{V})_{1}$.

The price of production now is calculated. The proof of this calculation is their realization in the commodities' exchange. To make this possible the price of production ' $\mathrm{V}_{1}$ has to be transformed into the market-price ' $\mathrm{U}_{0}$ by the price-form:
(13) ( ${ }^{\prime} V_{1}:={ }^{\prime} U_{0}$ ).

That's it.

## II

There are, as we have seen, two value-price transformations, that of value into price (8) and that of production-price into market-price (13). Both formulas are price-forms. The transformation problem - traditionally so-called - is a pseudo-transformation; in fact it is a value-value modification (Sweezy 1956, 54). This modification is a self-regulating cybernetic circle (10), but no problem.
In none of the inspected treatises on transformation is it understood that the form of value in general and the price-form in special is the solution. Anglo-Saxon authors tend to ignore the form of value, the most important discovery by Karl Marx (Blaug 1972, 225). But dialectics indeed are no matter of mathematical orientated simplicity. U and V, goods and value, are very different and we understand Mr. Samuelson, to whom values and prices are "mutually-exclusive alternatives" (1971, 400). For a practical man, of course, "Volume I's analysis of values" is an "unnecessary detour" (Samuelson 1971, 421) and does not serve anything to the techniques of price calculation. Marx was no pragmatist, but a very German thinker. Being a German, means to do something for its own sake. Marx's analysis of value therefore answers only that question, of what nature a value is. Being Hegel's good scholar Marx made his science out of a single notion: that
of commodity. All the following economic categories are motion-forms of value and usevalue, these "mutually-exclusive alternatives" which constitute the concepts of commodity, price-form, money, exchange, capital, and so on.
Mathematically orientated writers on Marx's economics prefer Marx-killing assumptions; if, for instance, all capital goods really had "the same unit turnover period" (VegaraCarrio 1974, 241), formula (10), the process of generalizing annual profit-rates, would be to one half unnecessary; it is nonsense to maintain, that labour be "measured in terms of unskilled or abstract labour" (Morishima 1973, 11), since labour is the cause of value, which is measured in use-values or units of the good that serves as price material or numeraire; values are not "determined only by technological coefficients" and they are not "independent of the market" (Morishima 1973, 15).
A term like $\mathrm{B}_{\mathrm{a}} \mathrm{p}_{\mathrm{b}}$ in Sraffa's equations $(1960,4)$ is an economic non-notion in Marx's sense: that quantity $B$ of commodity 'b' used up in the production of commodity 'a', multiplied by $\mathrm{p}_{\mathrm{b}}$, a unit's value of ' b ', results in a value, respectively price, but never contains the use-value. Marx is not on a Sraffa line; he described the motion-forms of commodity, the subject of his economics.

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